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ON GRAVITATION IN GASEOUS NEBULAE.

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In No. 3 of Vol. IX of these Transactions, appears a paper by C. M. Woodward, on the distribution of pressures and densities in a gaseous mass of constant temperature T_0, which is assumed to have a spherical form. At the surface of any sphere of radius R_0, whose center is at the center of mass, he finds the density and pressure to be

$$\delta_0 = \frac{CT_0}{2\pi k R_0^2}, \qquad (1)$$

$$P_0 = \frac{C^2 T_0^2}{2\pi k R_0^2}. \qquad (2)$$

The mass within the surface of radius R_0 Woodward found to be

$$M_0 = \frac{2CT_0R_0}{k}. \qquad (3)$$

The weight of a gramme at this surface is therefore

$$g_0 = k\frac{M_0}{R_0^2} = \frac{2CT_0}{R_0}. \qquad (4)$$

In these equations k is the gravitation constant, and C is the constant of the gas.

The values of these quantities at any sphere whose radius is R are also

$$\delta = \frac{CT_0}{2\pi k R^2}, \qquad (5)$$

$$P = \frac{C^2 T_0^2}{2\pi k R^2}, \qquad (6)$$

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$$M = \frac{2CT_0R}{k}, \tag{7}$$

$$g = \frac{2CT_0}{R}. \tag{8}$$

These equations are all consistent with each other. For example, as will be at once seen, the value of P_0 in (2) is

$$P_0 = \int_{R_0}^{\infty} g\delta dR, \tag{9}$$

where δ and g are given in (5) and (8). Also the value of M_0 in (3) is

$$M_0 = 4\pi \int_0^{R_0} R^2\delta dR. \tag{10}$$

It will be assumed for the purposes of discussion that $R < R_0$.

Let the entire mass contract, so that any mass originally within a sphere of any radius r_0, shall be within a radius r, and assume also that

$$\frac{r_0}{r} = \frac{R_0}{R} = \rho. \tag{11}$$

This means that the same law of density distribution shall prevail in the second state as in the first. Assume also that the temperature shall remain T_0 throughout the mass.

It is evident that the value of δ in (5) is the reciprocal of v, where v is the volume of unit mass and that from (5) and (6) the product

$$Pv = P \times \frac{1}{\delta} = CT_0. \tag{12}$$

The same equation may also be applied to the entire sphere in its initial condition. Let V_0 be its volume. Its average density is three times the density (5) at the surface. Then when applied to the entire sphere by (3) and (2) the equation becomes

$$3P_0V_0 = \frac{3C^2T_0^{\,2}}{2\pi kR_0^{\,2}}\,\frac{4}{3}\,\pi R_0^{\,3} = CT_0M_0. \qquad (13)$$

This result agrees perfectly with (12), that equation applying to volume of unit mass, and (13) to the volume corresponding to the average density of mass M_0.

The same equation applied to the mass M_0 when contracted to volume $V = \frac{4}{3}\pi R^3$ gives

$$3PV = P \times 4\,\pi R^3 = CT_0M_0. \qquad (14)$$

Hence the pressure which must be applied to the contracted sphere, in order to hold it in equilibrium, is, by solving (14) and replacing M_0 from (3),

$$P = \frac{C^2T_0^{\,2}R_0}{2\pi kR^3} = \frac{C^2T_0^{\,2}\rho}{2\pi kR^2}. \qquad (15)$$

The pressure at this surface after contraction has taken place, is therefore ρ times as great as was required in the initial state, as will be seen from (6).

The weight of unit mass at the surface after contraction is,

$$g = k\frac{M_0}{R^2} = \frac{2CT_0R_0}{R^2} = \frac{2CT_0\rho}{R}. \qquad (16)$$

This value is also ρ times as great as the value given by (8).

It is evident therefore that the weight of a gramme at any fixed point within the entire mass has, by reason of this shrinkage, been multiplied by ρ.

It follows that the density of the gas at the surface R, where the pressure is now given by (15), must be

$$\delta = \frac{P}{CT_0} = \frac{CT_0\,\rho}{2\pi kR^2}. \qquad (17)$$

The effect of this shrinkage upon the pressure which the superposed layers exert upon the sphere will now be determined.

That pressure is

$$P = \int_R^\infty g\delta dR. \tag{18}$$

As both g and δ have by the shrinkage been multiplied by ρ, it follows that P in (6) must have been multiplied by ρ^2. Putting in the values of g and δ from (16) and (17) the pressure due to gravitation, of these superposed layers, is on integration

$$P = \frac{C^2 T_0^2 \rho^2}{2\pi k R^2}. \tag{19}$$

It is therefore evident that, on account of skrinkage due to gravitation, the gravitating pressure exerted radially inward across any and every concentric spherical surface has become ρ times as great as the internal mass can support, unless the temperature has increased. When he comes to deal with the subject of contraction, Woodward seems to have omitted from his analysis the work done in compressing unit mass, due to increase in weight of superposed layers. At any fixed point in space, the weight of a gramme increases as shrinkage proceeds, because the mass of matter internal to it is increasing. The work *pdv* done on unit mass in the case discussed by him, does not all come from the action of gravitation upon the unit mass itself. As a result Woodward not only rejects the work of See, but Lane's law as well.

For the initial conditions the equation for a perfect gas gives,

$$\frac{P_0}{\delta_0} = CT_0. \tag{20}$$

Evidently in the second condition of equilibrium

$$\frac{P_0 \rho^2}{\delta_0 \rho} = C\rho T_0 = CT. \tag{21}$$

where

$$T = \frac{r_0 T_0}{r}. \tag{22}$$

It is evident that (7) with T variable is an expression of the law involved in (22) and which was first announced by A. Ritter* in the form

$$Tr = T_0 r_0 = \text{constant}.$$

According to (7)

$$TR = \frac{kM}{2C}, \qquad (23)$$

where k is the gravitation constant whose value is given below, C is the constant for the kind of gas (PvT^{-1}), and M is the mass of the gravitating gas within radius R from the center of mass.

Equation (23) is in a form which makes it available for inquiry into cosmical problems.

It may be of interest to apply these equations to an infinite mass of gas, having the physical constants which hydrogen has under ordinary conditions such as we can supply at ordinary temperatures. Imagine the central core of this mass, to have within a radius equal to that of our sun, a mass equal to that of the sun.

The quantities thus given are

$M = 1.842 \times 10^{33}$ grammes.
$R = 6.972 \times 10^{10}$ cm.
$\frac{1}{k} = 1.543 \times 10^{7}$ in C. G. S. units.
$C = 4.143 \times 10^{7}$ "

By equation (23)

$$T = \frac{Mk}{2CR} = 20{,}675{,}000^\circ \text{ C}.$$

This temperature is certainly much greater than that of the sun.

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* Ann. der Phys. u. Chemie, Bd. V, 1878, S. 546.

Putting this temperature in (5), the density at the surface of this sphere is

$$\delta = \frac{CT}{2\pi k R^2} = 0.432.$$

The density of hydrogen under standard conditions (0°,760^{mm}) is 0.000089578.

The pressure at the surface due to the superposed layers is, by (6),

$$P = \frac{C^2 T^2}{2\pi k R^2} = 3.706 \times 10^{14}.$$

This is 366,000,000 atmospheres. The density at the distance of one-tenth radius from the center would therefore be 43.2 times that of water at maximum density and the pressure would be 3.66×10^{10} atmospheres. The density at a distance of 92 million miles from the center of mass would, however, 20 be 0.0000376 if the temperature of all space were ~~ten~~ million degrees C. This is about four-tenths the density of hydrogen under standard conditions.

The average density of the gaseous sphere being three times the density at the surface, the average density of the solar hydrogen sphere is 1.296. The average density of the sun itself is about 1.39, or about 7 per cent. greater than that of the hydrogen sphere.

The density of hydrogen gas near the center of the sphere here discussed would probably be much greater than that of solid hydrogen under a pressure of one atmosphere and at its fusing point. Nevertheless it is a gas, for its temperature is far above the critical temperature of hydrogen.

If a sample of it could be collected in a boiler, of adequate strength and infusibility, and it were brought into our interplanetary space, in the shadow of some planet, it would cool to the freezing point of hydrogen. Even then the pressure on the boiler might be enormous. If the boiler be now enlarged by means of a telescope joint until this pressure is taken off, the boiler may be opened. It will be full of solid hydrogen. The pressure of its vapors will, of course, be very much smaller than would be the case in the atmosphere of the hydrogen sun. In fact the pressure of gases of all kinds in interplanetary space must be practically obliterated, by reason of the low temperature. The facilities for refrigeration on the outskirts of our universe seem to be very effective. And this may also serve to aid in understanding the conditions existing on the sun.

If the gaseous mass here discussed were initially under isentropic conditions, it would remain in equilibrium. It could not contract, nor could its temperature change. If it be assumed to lose heat with equal facility from all parts, it will contract and the temperature will rise uniformly throughout. The greater the heat loss per second, the more rapidly will the mass pass through its history of condensation. If heat escapes from the denser parts near the core, with less facility than from the less dense parts, then the temperature throughout the mass at any instant may become a function of the radius. This seems to be the condition around our own sun.

A gas having greater density than hydrogen, under standard conditions, would have a proportionately smaller constant C.

The temperature computed above for the hydrogen sun would be proportionately greater than 20,000,000 degrees. Equation (23) in fact shows that for a fixed radius R of spherical core, in such a gaseous mass, and containing a given internal mass M, the product $T\ C$ is constant for all gases. Hence, δ and P at the surface of the sphere, computed numerically for the hydrogen mass, would have the same numerical values for any other gas, under the conditions which this entire discussion assumes.

The average density of the entire spherical mass would then be independent of the nature of the gas, and would be 1.296 grammes per cubic centimeter, as was above shown.

The conditions surrounding our own sun do not appear to differ in a way that can negative the conclusion that, in order to account for its increased average density and low temperature, some liquid condensation may be assumed to have already taken place. With the law of gase-

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ous distribution here used as a basis of comparison, the density of the gas at the center is infinite, and still the average density of the sun could not exceed 1.296 under conditions of stability, if it were wholly gaseous, and the pressure of 366,000,000 atmospheres were superposed upon it, the temperature being 20,000,000 degrees at least. Of course, if the sun has a liquid core, it must be composed of substance whose critical temperatures are above that of the sun.

The low temperature of the sun seems to be almost wholly compensated, by the obliteration of cosmical or interplanetary pressure.

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